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Bound states in modified action for light quarks in instanton vacuum model

Alexander E. Dorokhov [a,b], Sergey V. Esaybegyan [c,d,*]

[a] Joint Institute for Nuclear Research, Bogoliubov Laboratory of Theoretical Physics, 114980, Dubna, Russia
[b] Institute for Theoretical Problems of Microphysics, Moscow State University, RU-119899, Moscow, Russia
[c] Argosy University/Twin Citis. 1717, Central Parkway, Eagan, MN 55121, USA
[d] Yerevan Physics Institute, Armenia

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ABSTRACT

Within the zero fermion mode approximation and using the instanton liquid action modified by the effective determinant interaction we are able to describe low-lying bound states in the pseudoscalar channel for the pion as well in the vector channel for the ρ-meson.

1. Introduction

In this Letter, we will show that instanton fluctuations of the QCD vacuum not only describe the mechanism of spontaneous breaking of chiral symmetry (SBCS), but also can play a decisive role in formation of bound states in the vector channel. It should be emphasized that both channels (pseudoscalar and vector) can be described using a unified approach based on the quark interaction generated by instantons.

Earlier, in series of papers [1–10], it was shown that large-scale instanton fluctuations of the QCD vacuum can explain the mechanism of SBCS. This mechanism is based on the idea [4] of mixing and delocalization of quark zero modes in the field of instantons I and anti-instantons $\bar{I}$. A model of the QCD vacuum as the instanton–anti-instanton $I\bar{I}$ liquid has been constructed and applied to many problems of low energy hadronic physics [1–16]. It treats the vacuum state as a discrete set of $I\bar{I}$ pairs and transition to the continuous limit is taken in the final stage of the calculations. In such picture there is no dependence on the order of how the thermodynamic limit is taken ($N \to \infty$, $V \to \infty$, $N/V = \text{const}$, where N is the number of pseudoparticles, and V is the 4-volume). The propagation of a quark in an instanton vacuum leads to appearance of an effective momentum dependent quark mass $M(p)$ and the pion appears as a Goldstone boson. In this case a fundamental role is played by the interaction of quarks with vacuum.

However, in an analysis of non-Goldstone modes, e.g., vector particles, the exchange of momentum between quarks is of principle importance in order to produce a bound state. This effect becomes possible if consider the instanton vacuum as a continuous medium. The problem of dealing correctly with both effects, the interaction of the quark with large-scale instanton fluctuations of vacuum and the interaction of quarks with each other, can be solved by taking an expectation value of the Lagrangian of quantum chromodynamics (QCD) in a statistical ensemble of pseudoparticles, rather than of individual correlation functions, as it was done in [4,6,7,10]. Such approach was partly developed in earlier papers [17,18] and reinvestigated recently in [19]. In these works it was demonstrated that after an averaging the effective action is modified [19] as

$$\begin{aligned} S = {} & \int \frac{d^4k}{(2\pi)^4} \left(\psi^+\left[\hat{k} - iM(k)\right]\psi + \chi^+\hat{k}\chi\right) \\ & + 2\frac{V}{N}\int \frac{d^4k\, d^4p\, d^4q\, d^4l}{(2\pi)^{12}} \left(M(k)M(q)M(p)M(l)\right)^{1/2} \\ & \times \delta^4(k+p-q-l)\Big\{\left(\psi_L^+(k)\chi_L(q)\right)\left(\chi_L^+(p)\psi_L(l)\right) \\ & + 2\frac{V}{N}\Big[\left(\psi_{1L}^+(k)\psi_{1L}(q)\right)\left(\psi_{2L}^+(p)\psi_{2L}(l)\right) \\ & - \left(\psi_{1L}^+(k)\psi_{2L}(q)\right)\left(\psi_{2L}^+(p)\psi_{1L}(l)\right)\Big]\Big\} + (L \to R), \end{aligned} \tag{1}$$

where the case $N_f = 2$ is considered. The first term in the interaction part of Eq. (1) describes momentum transfer between quark due to exchange the massless bosonic spinor (ghost) fields χ, χ^+ and, therefore, it is responsible for the formation of bound state. The second and third terms in the interaction part of Eq. (1)

* Corresponding author at: Argosy University/Twin Citis. 1717, Central Parkway, Eagan, MN 55121, USA.
E-mail addresses: dorokhov@theor.jinr.ru (A.E. Dorokhov), sesaybegyan@argosy.edu (S.V. Esaybegyan).

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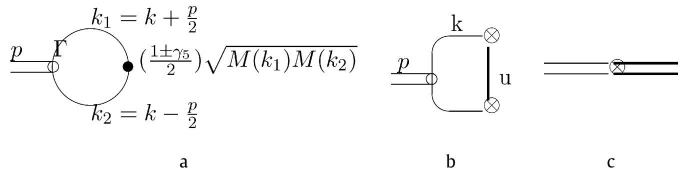

Fig. 1.

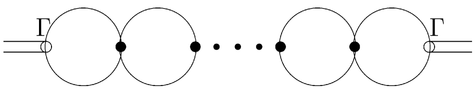

Fig. 2.

describe the four-fermion interaction with large scale vacuum fluctuation. Note, that the strength of this 4-fermion interaction differs from the results of [10] (see [19]).

In this Letter we are going to demonstrate that the effective action (1) leads to reasonable description of low-lying bound states in pseudoscalar (Section 2) and vector (Section 3) channels.

2. Pseudoscalar channel

In this section, we consider the correlation function for pseudoscalar currents and demonstrate, following the standard procedure of [4,10], that it is possible to describe the π-meson as a Goldstone boson. We present these calculations for completeness and also in order to prove that the modified effective action (1) reproduces the well-known results.

The nonperturbative vacuum filled with quark pairs in the $^3P_0$ state and is characterized by nontrivial quark condensate

$$\langle\bar\psi\psi\rangle = i\,\mathrm{tr}\,S(x,x) = i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\,S(p) = -4N_c\int\frac{d^4p}{(2\pi)^4}\frac{M(p)}{p^2+M^2(p)},$$

where it should be understood the relation between Minkowski and Euclidean definitions as $\langle\bar\psi\psi\rangle_M = -i\langle\psi^+\psi\rangle_E$.

If a quark pair is reconstructed as $^3P_0 \to {}^1S_0$ by using an external probe (hadronic current), that does not introduce substantial changes in the system (Fig. 1a), then it leads to formation of the 0^- state (pion). Zero mass of the pion in the chiral limit implies that this rearrangement does not disturb the vacuum at all and does not change the energy of fluctuations (see, for example, [20]). The four-quark interaction terms in the effective action (1) are precisely responsible for these effects.

We define the connected part of the correlation function in different channels as usual

$$\Pi^\Gamma(p) = \int\frac{d^4k_1d^4k_2}{(2\pi)^4}\delta(k_1-k_2+p)J^\Gamma(k_1)J^\Gamma(k_2)e^{-S},$$
$$\text{where } \Gamma = 1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu}, \qquad (2)$$

S is for the action (1) and J is for the currents. The effective quark–meson vertices $M \to q\bar q$ corresponding to the effective action (1)[1] are denoted as $\Gamma(p)$ for the four-quark interaction (Fig. 1a), $\Gamma_\mu(u,p)$ for the quark interaction in the vector channel due to exchange by bosonic spinor fields (Fig. 1b), where a cross is the first term in (1), and in Fig. 1c thin lines are quarks, thick lines are bosonic spinor fields.

In the pseudoscalar channel ($\Gamma = \gamma_5$), the leading in $1/N_c$ expansion contribution to the correlation function Π^{γ_5} is represented by the chain diagrams shown in Fig. 2. Solid thin lines correspond to propagating quarks, while bold dot is for the four-quark effective interaction.

$$\begin{aligned}\Gamma^{\gamma_5}_\pm(p) &= \int\frac{d^4k}{(2\pi)^4}\frac{\sqrt{M_1M_2}}{[(k+p/2)^2+M_1^2][(k-p/2)^2+M_2^2]}\\ &\quad\times\mathrm{Tr}\left[\gamma_5(\hat k+p/2+iM_1)\frac{1\pm\gamma_5}{2}(\hat k-p/2+iM_2)\right]\\ &\simeq \mp 2\int\frac{d^4k}{(2\pi)^2}\frac{M(k)}{k^2+M^2} = \pm\frac{\langle\bar\psi\psi\rangle}{2N_c},\end{aligned} \qquad (3)$$

where $M_i \equiv M(k_i)$. The four-quark vertex gives the dominant contribution in (3), while the term in (1) related to the ghost field is suppressed by the packing fraction factor ($\rho/R \simeq 1/3$) proportional to the instanton liquid density ($\rho \sim (600\ \text{MeV})^{-1}$ is the average size of instantons and R is the inter-instanton distance, $R^{-1} \simeq 200$ MeV). Note, that the vertex function shown in Fig. 1b vanishes for $\Gamma = \gamma_5$ and in the one-flavor case, $N_f = 1$, produces a zero contribution (see (1)) in agreement with the results of [5]. From symmetry considerations it follows, that the vertex $\Gamma^\Gamma_\pm(p)$ (Fig. 1a) vanishes in the vector ($\Gamma = \gamma_\mu$) and tensor ($\Gamma = \sigma_{\mu\nu}$) channels, that is also in accordance with earlier results [10].

By using $\Gamma_\pm$ defined in Eq. (3), the connected part of the correlation function in the pseudoscalar channel is represented as

$$\begin{aligned}\Pi_{con} &= -2N_c^2\frac{2V}{N}\Gamma_\pm(p)\left[1+\left(\frac{2VN_c}{N}\right)(N_cL)\right.\\ &\quad\left.+\left(\frac{2VN_c}{N}\right)^2(N_cL)^2+\cdots\right]\Gamma_\mp(p)\\ &= -\frac{4VN_c^2}{N}\Gamma_\pm(p)\frac{1}{R_-(p)}\Gamma_\mp(p),\end{aligned} \qquad (4)$$

where

$$R_-(p) = 1-\frac{2VN_c}{N}L$$

and the quark loop L is

$$\begin{aligned}\frac{2VN_c}{N}L &= \frac{2VN_c}{N}\int\frac{d^4k_1d^4k_2}{(2\pi)^4}\delta^4(p+k_1-k_2)M_1M_2\\ &\quad\times\mathrm{Tr}\left[\frac{\hat k_1+iM_1}{k_1^2+M_1^2}\left(\frac{1\pm\gamma_5}{2}\right)\frac{\hat k_2+iM_2}{k_2^2+M_2^2}\left(\frac{1\mp\gamma_5}{2}\right)\right]\\ &= \frac{4VN_c}{N}\int\frac{d^4k_1d^4k_2}{(2\pi)^4}\delta^4(p+k_1-k_2)\\ &\quad\times M_1M_2\frac{(k_1k_2)}{(k_1^2+M_1^2)(k_2^2+M_2^2)}+O(\rho/R).\end{aligned} \qquad (5)$$

Using the self-consistency condition [4,19]

$$\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\frac{M^2(p)}{p^2+M^2(p)} = 1, \qquad (6)$$

[1] All factors N_c and $\frac{N}{V}$ are included in the correlation function.

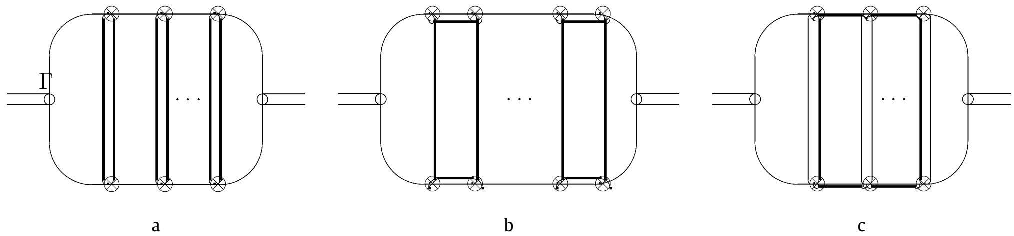

Fig. 3.

$R_-(p)$ is reduced to

$$R_-(p) = \frac{2VN_c}{N}\int \frac{d^4k_1 d^4k_2}{(2\pi)^4}\delta^4(p+k_1-k_2)\frac{(M_1k_{2\mu}-M_2k_{1\mu})^2}{(k_1^2+M_1^2)(k_2^2+M_2^2)}$$
$$\simeq p^2\frac{2VN_c}{N}\int \frac{d^4k}{(2\pi)^4}\frac{M^2}{(k^2+M^2)^2} = \beta p^2. \tag{7}$$

Bringing together all terms for the connected part of the correlation function in the pseudoscalar channel, one gets

$$\Pi^5(p) = N_c\frac{VN_c}{N}\left[\frac{\langle\bar\Psi\Psi\rangle}{N_c}\right]^2\frac{1}{\beta p^2}.$$

Using the well-known asymptotic in the limit of small p relation $\Pi^5(p) = 4\langle\bar\Psi\Psi\rangle^2/f_\pi^2p^2$,

following from the current algebra (see, for example, [4]), one obtains for the pion decay constant

$$f_\pi^2 = 4\beta\frac{N}{V} \simeq 8N_c\int\frac{d^4k}{(2\pi)^4}\frac{M^2(k^2)}{(k^2+M^2(k^2))^2}. \tag{8}$$

The same result follows from the diagrammatic analysis [4]. Recall, that the effective mass $M(k)$ is parametrically small in the packing fraction of medium ($M(0)\sim\rho/R^2$). The scale, at which $M(k)$ changes, is $1/\rho$, therefore, the integral in (8) determined by a broad range of parametrically small momenta $1/R \ll k \ll 1/\rho$. Calculating (8) with logarithmic accuracy, we obtain

$$f_\pi^2 \cong \frac{N_c}{\pi^2}M^2(0)\ln\frac{1}{M(0)\rho}. \tag{9}$$

We see, that f_π is parametrically small as compared to characteristic hadron scale, the latter being determined in the instanton vacuum by the average size of pseudoparticles. Thus, the experimental smallness of $f_\pi = 132$ MeV is naturally explained within the instanton vacuum model [4]. Numerically, according (8), $f_\pi = 142$ MeV. Thus, we showed that our approach yields the results for the pseudoscalar channel in accordance with the well-known results.

3. Vector channel

Let us recall the usual arguments leading to zero result for the correlation function in the vector channel in the instanton model involving the vacuum as an ensemble of $I\bar I$ pairs. The first kind of arguments is associated with the chiral properties of the quark propagator [6], i.e. two quarks and two antiquarks generated by external currents cannot directly form the bound state in the vicinity of the same pseudoparticle. In the model with modified effective action (1), this type of diagrams corresponds to the contributions ($\Gamma^\mu_\pm = 0$) (see Fig. 1a) vanishing in the vector channel in the chiral limit.

The second kind of arguments is associated with the color symmetry. It is well known that the instanton solution in QCD is a vector of the $SU(2)$ subgroup of the color group $SU(3)$. Since the G-parity type transformation in the color space, i.e. the product of charge transformation and the 180° rotation around the second axis in the $SU(2)$ subgroup of the color group, does not alter the instanton field, the zero result for the vector channel correlation function follows from the fact that the vector current is odd under this transformation [21]. All above considerations imply that rescattering by pseudoparticles forms a discrete series of events. This would obviously be the case in our model as well if we took into account only the four-fermion terms in the action (1).

The situation radically changes when exchanges produced by the quark–ghost term in the modified action (1) are included. Due to effects of permanent rescattering in continuous instanton media (Fig. 3), a nonzero contribution (first term in (1)) to the vector channel is obtained in our approach.

If, however, we isolate only one-meson states in the correlation function by imposing the condition that any structure obtained by cutting an arbitrary diagram contains only two quarks and that there are no colorless subsystems formed by ghost legs (in other words, if we recall the general rule, that only planar diagrams and minimum number of quark loops survive in the limit $N_c\to\infty$ [20]), the class of diagrams presented in Fig. 3c can be discarded. This corresponds to neglect of the contribution of continuum to the correlation function.

In leading in the $1/N_c$ approximation, the correlation function is thus determined by the diagrams of Figs. 3a and 3b. They can be summed in the standard way by using the Fredholm equation. In the vector channel the polarization function $\Pi_{\mu\nu}$ from Eq. (2) is given by

$$\Pi_{\mu\nu} = \int\frac{d^4k_1d^4k_2}{(2\pi)^4}\delta(k_1-k_2+p)J_\mu(k_1)J_\nu(k_2)e^{-S}$$
$$= 2N_c\lambda\int\frac{d^4u}{(2\pi)^4}\Gamma_\mu(u,P)\Gamma_\nu(u,P)$$
$$+ 2N_c\lambda^2\int\frac{d^4ud^4v}{(2\pi)^8}\Gamma_\mu(u,P)R(u,v)\Gamma_\nu^+(v,P), \tag{10}$$

where $\lambda = 4V^2N_c/N^2$. The vertex $\Gamma_\mu(u,P)$ is defined in Fig. 1b:

$$\Gamma_{\mu^\pm}(u,P) = \int\frac{d^4k}{(2\pi)^4}\sqrt{M(k)M(k+P)}M(u+k)$$
$$\times\,\mathrm{Tr}\left(\frac{\hat k+iM}{k^2+M^2}\gamma_\mu\frac{\hat k+\hat P+iM}{(k+P)^2+M^2}\frac{1\mp\gamma_5}{2}\right.$$
$$\left.\times\frac{\hat u+\hat k}{(u+k)^2}\frac{1\pm\gamma_5}{2}\right), \tag{11}$$

where $\Gamma_{\mu^+} = \Gamma_{\mu^-} = \Gamma_\mu$ and the function $R(u,v)$ satisfies to the Fredholm equation

$$R(u,v) = K(u,v) + \lambda\int\frac{d^4k}{(2\pi)^4}K(u,k)R(k,v). \tag{12}$$

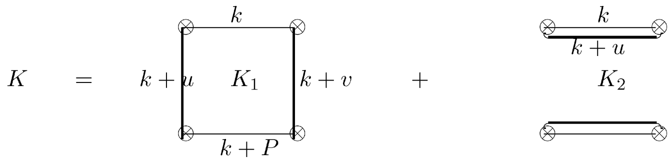

Fig. 4.

The kernel $K(u,v)$, shown in Fig. 4, is given by

$$K_1(u,v) \simeq \int \frac{d^4k}{(2\pi)^4} M(k)M(k+P)M(u+k)M(v+k) \times \mathrm{Tr}\left(\frac{\hat{k}}{k^2+M^2}\frac{1\pm\gamma_5}{2}\frac{\hat{u}+\hat{K}}{(u+k)^2}\frac{1\mp\gamma_5}{2} \times \frac{\hat{k}+\hat{P}}{(k+P)^2+M^2}\frac{1\pm\gamma_5}{2}\frac{\hat{v}+\hat{k}}{(v+k)^2}\frac{1\mp\gamma_5}{2}\right), \quad (13)$$

and

$$K_2(u,v) \simeq N_c(2\pi)^4\delta^4(u-v) \times \left(\int \frac{d^4k}{(2\pi)^4} M(k)M(u+k)\,\mathrm{Tr}\,\frac{\hat{k}}{k^2+M^2}\frac{1\pm\gamma_5}{2} \times \frac{\hat{u}+\hat{k}}{(u+k)^2}\frac{1\mp\gamma_5}{2} \times (u\to v)\right), \quad (14)$$

where additional factor N_c in K_2 is related to the number of loops, which is even in K_2. We perform the calculations in the leading order approximation in the packing parameter ρ/R. In this approximation one has $\rho M \ll 1$, $\rho u \ll 1$ and $\rho v \ll 1$. Since $M(k)$ is a rapidly decreasing function as $k^2 \to \infty$ (see [4]), in the leading logarithmic approximation it is possible to substitute $M(k_i) \to M(0)$ in K_1 and introduce a cutoff parameter in the upper limit of the integral, $|k| \leqslant 1/\rho$ (see discussion after (8)).

At vanishing external momentum of the particle $P \to 0$, we find for the kernel $K(u,v)$

$$K(u,v) = K_1(u,v) + K_2(u,v),$$
$$K_1(u,v) \simeq 2M^4(0)\int \frac{1}{(u+k)^2(v+k)^2}\frac{d^4k}{(2\pi)^4} \simeq -\frac{M^4(0)}{8\pi^2}\ln\rho^2(u-v)^2,$$
$$K_2(u,v) \simeq (2\pi)^4\delta^4(u-v)N_c\left(\int \frac{2M^2(k)}{k^2+M^2}\frac{d^4k}{(2\pi)^4}\right)^2 \simeq \frac{1}{\lambda}(2\pi)^4\delta^4(u-v), \quad (15)$$

where the self-consistency condition (6) and the definition for λ (10) are used.

In calculation of $\Gamma_\mu(u,P)$ as $P \to 0$, note that in the leading approximation the γ_5 term vanishes (due to symmetry consideration) and, in order $up/p^2 \sim 1$, we keep only terms:

$$\Gamma_\mu(u,P)|_{P\to 0} \simeq \frac{M^2(0)}{16(\pi)^4}(\gamma_{1\mu}+\gamma_{2\mu}+\gamma_{3\mu}+\gamma_{4\mu}),$$
$$\gamma_1 \simeq \int d^4k \frac{2(P_\mu-u_\mu)k^2}{(k^2+M^2)[(k+P)^2+M^2](u+k)^2} \simeq \pi^2(2u_\mu-P_\mu)\ln\left(\rho^2u^2+\rho^2M^2\right)+O(\rho/R),$$
$$\gamma_2 \simeq \int d^4k \frac{(2k_\mu+P_\mu)}{(k^2+M^2)^2} \simeq O(\rho/R),$$
$$\gamma_3 \simeq \int d^4k \frac{(-2u_\mu)}{(k^2+M^2)(u+k)^2} \simeq \pi^2u_\mu\ln\left(\rho^2u^2+\rho^2M^2\right)+O(\rho/R),$$
$$\gamma_4 \simeq \int d^4k \frac{2u^2(\frac{Pu}{u^2}-1)k_\mu-u^2P_\mu}{(k^2+M^2)[(k+P)^2+M^2](u+k)^2} \simeq \pi^2\frac{2}{3}P_\mu\frac{Pu}{u^2}\ln\frac{M^2}{M^2+u^2}+O(\rho/R),$$
$$\Gamma_\mu(u,P)|_{P\to 0} \simeq \frac{M^2(0)}{16\pi^2}\left[(2u_\mu-P_\mu)\ln\left(\rho^2u^2+\rho^2M^2\right) + \frac{2}{3}P_\mu\frac{Pu}{u^2}\ln\frac{M^2}{M^2+u^2}\right]. \quad (16)$$

Since the kernel K is a function of the difference $(u-v)$, Eq. (12) can be solved by using the Fourier transformation:

$$R(x) = \tilde{P}\frac{K(x)}{1-\lambda K(x)}, \quad (17)$$

where $\tilde{P}$ means the principal value, and

$$\Pi_{\mu\nu}(P) = 2N_c\lambda\int d^4x\,\Gamma_\mu(P,x)\Gamma_\nu(P,x) + 2N_c\lambda^2\int d^4x\,\Gamma_\mu(P,x)R(x,P)\Gamma_\nu^+(x,P). \quad (18)$$

After Fourier transformation one has the following expressions for $K_1(x)$ and $\Gamma_\mu(x,P)$:

$$K_1(x) = -\frac{M^4}{8\pi^4}\Phi(x),$$
$$\Phi(x) = \frac{1}{x^4}\left[J_0\left(\frac{|x|}{\rho}\right)+\frac{|x|}{2\rho}J_1\left(\frac{|x|}{\rho}\right)-1\right], \quad |x| = \sqrt{x^2},$$
$$\Gamma_\mu(x,P) = \frac{M^2}{16\pi^2}\int\frac{d^4u}{(2\pi)^4}e^{iux}\left[\left(2i\frac{x_\mu}{|x|}\frac{d}{d|x|}-P_\mu\right) \times \ln\left(\rho^2u^2+\rho^2M^2\right)+\frac{2}{3}P_\mu\frac{Px}{u^2}\frac{i}{|x|}\frac{d}{d|x|} \times \ln\frac{\rho^2M^2}{\rho^2(u^2+M^2)}\right]$$
$$= \frac{M^2}{(2\pi)^4}\left[2ix_\mu\Phi_1(x)-P_\mu\Phi(x) + \frac{i}{3}P_\mu(Px)\left[\Phi(x)\ln\left(\rho^2M^2\right)+\Phi_2(z,x)\right]\right],$$
$$\Phi_1(x) = \frac{1}{|x|}\frac{d}{d|x|}\Phi(x),$$
$$\Phi_2(z,x) = \frac{1}{|x|}\frac{d}{d|x|}\left[\frac{1}{x^2}\int_0^1\left[1-J_0\left(z\frac{|x|}{\rho}\right)\right]\frac{z\,dz}{z^2+\rho^2M^2}\right], \quad (19)$$

where $J_i(z)$ are the Bessel functions of the first kind. Note, that the integrand in $\Phi_2(z,x)$ is peaked near the upper limit $(z=1)$ and, within the leading in ρ/R approximation, $\Phi_2(z,x) \simeq 2\Phi(x)$. Using (19), we find the following expression for the correlation function in the vector channel as $P \to 0$:

$$\Pi_{\mu\nu}(P) = 2N_c\lambda\tilde{P}\int d^4x\,\frac{\Gamma_\mu(P,x)\Gamma_\nu^+(P,x)}{1-\lambda K(x)} = -2N_c\tilde{P}\int d^4x\,\frac{\Gamma_\mu(P,x)\Gamma_\nu^+(P,x)}{K_1(x)}$$

$$= -\frac{N_c}{16\rho^2\pi^2}\left[(I_0 - I_1)\delta_{\mu\nu} + I_2\rho^2 P_\mu P_\nu\right],$$

$$I_0(a) = \frac{8Q(a)}{a^2}, \qquad I_1(a) = \tilde{P}\int_0^a \frac{zJ_2^2(z)\,dz}{Q(z)},$$

$$I_2(a) = A(a) + 2/3(\ln\rho M + 1)4A(a) + Q(a), \tag{20}$$

where $Q(z) = 2J_0(z) - 2 + J_1(z)z$, $A(a) = 2A_1(a) + J_0(a) - 1$ with $a = \frac{|x|_{\text{upper limit}}}{\rho}$, and $A_1(a) = \int_0^a \frac{1-J_0}{z}\,dz$.

It is well known [22], that, in the limit $P \to 0$, the Kallen–Lehmann representation integral is dominated by one-particle states. Thus, the correlation function in this limit becomes (in Euclidean space)

$$\Pi_{\mu\nu}(P) = -\Sigma\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{m_i^2}\right)f_i^2. \tag{21}$$

The current matrix element is given by: $\langle 0|\bar{d}\gamma_\mu u|\rho\rangle = \epsilon_\mu^\lambda f_\rho m_\rho$, where $f_\rho^{exp} = 200$ MeV and ϵ_μ is the polarization vector of the ρ-meson. Taking into account that f_i are very small for excited states [23] $(\rho' \cdots)$, we finally can write

$$\Pi_{\mu\nu}(P) = -\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{m_\rho^2}\right)f_\rho^2. \tag{22}$$

Comparing (20) and (22), we find

$$f_\rho^2 = \frac{N_c}{16\rho^2\pi^2}(I_0 - I_1), \qquad m_\rho^2 = \frac{1}{\rho^2}\frac{(I_0 - I_1)}{I_2}. \tag{23}$$

In other words, both the constant f_ρ and the mass m_ρ are inversely proportional to the average size of instanton ($\frac{1}{\rho} = 600$) MeV. For numerical estimates we choose the cut-off parameter a in the interval $5 < a \leqslant 10$ corresponding to distances much bigger than the average instanton radius. Within this interval we get for $a = 5$, $f_\rho = 57$ MeV, $m_\rho = 182$ MeV and for $a = 10$, $f_\rho = 141$ MeV, $m_\rho = 407$ MeV. These results are in qualitative agreement with experimental values: $f_\rho = 193$ MeV, $m_\rho = 776$ MeV.

4. Conclusions

In this Letter we demonstrated that proper inclusion of the quark interaction induced by instantons allow us to go beyond the quenched approximation, accuracy of which is of order $N_f/N_c = 2/3$. The modified effective action makes it possible to describe the pseudoscalar channel with very good accuracy, while the low-lying bound state in vector channel, ρ-meson, is described within 30% accuracy. We need to bear in mind that the assumptions made in the vector channel introduce an error of order 30%–40% in these estimates and require further improvements in the model calculations.

Let us mention the results of [24], where the vector meson was considered, and also [25,26], where the vector correlator functions was considered, within the instanton liquid model. In [24] it was found, that the vector mesons have the masses about 30% larger than the experimental values. The authors of [24] stress in their conclusions that the confinement is very important in determination of the resonance widths and its inclusion can improve an agreement. Recall, that our results show the value for the ρ-meson mass that is 30% smaller. In our opinion, the disagreement with the experimental value could be relaxed, if we include in our model the current quark masses. We would like to emphasize that in our approach the vector channel has a nonzero contribution from the instanton induced quark interaction due to effects of permanent scattering in a continuous medium. This interaction is attractive and becomes responsible for formation of ρ-meson as a bound state.

Finally note, that two different terms in the interaction of (1) have different physical meaning. The four-quark interaction term is responsible for the onset of a Goldstone mode. The quark–ghost interaction term in (1) makes it possible to describe a momentum transfer between dynamical quarks and, therefore, responsible for the formation of bound states in non-Goldstone modes. Possibly, this is similar to strong effective interaction between quarks, which was found in [27]. Quark interaction in the instanton medium, in our approach, is mediated by spin one-half boson ghost fields χ and χ^+ corresponding to the effective degrees of freedom of the continuous medium.

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